

# Chapter 5

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## The laws of motion

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29-Sep-25

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# Lecture 04

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## Forces Of Friction

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## Forces of Friction

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When an object is in motion on a surface or through a viscous medium, there will be a resistance to the motion.

This resistance is called the force of friction.

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## Forces of Friction

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Friction is proportional to the normal force.

- $f_s \leq \mu_s n$
- $f_k = \mu_k n$

These equations relate the magnitudes of the forces; they are not vector equations.

For static friction, the equals sign is valid only when the surfaces are on the verge of slipping.

Use the inequality for static friction if the surfaces are not on the verge of slipping.

$\mu$  is the coefficient of friction

- $\mu_s > \mu_k$
- $0 < \mu_s \leq 1$
- $0 < \mu_k < 1$

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## Forces of Friction

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The coefficient of friction depends on the surfaces in contact.

The force of static friction is generally greater than the force of kinetic friction.

The direction of the frictional force is opposite the direction of motion and parallel to the surfaces in contact.

The coefficients of friction are nearly independent of the area of contact.

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## Static Friction

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Static friction acts to keep the object from moving.

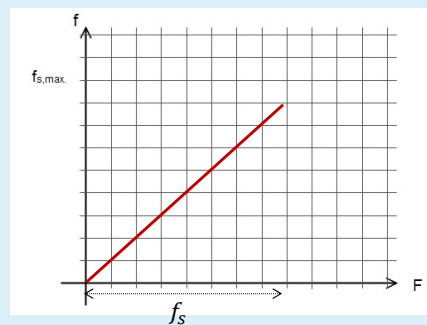
As long as the object is not moving,  $f_s = F$

If  $\vec{F}$  increases, so does  $\vec{f}_s$

If  $\vec{F}$  decreases, so does  $\vec{f}_s$

$$f_{s,max} = \mu_s n$$

The equality holds when the surfaces are on the verge of slipping.



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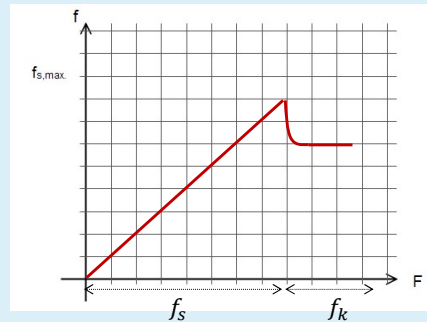
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## Kinetic Friction

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The force of kinetic friction acts when the object is in motion.

$$f_k = \mu_k n$$



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## Some Coefficients of Friction

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**TABLE 5.1** Coefficients of Friction

|                             | $\mu_s$  | $\mu_k$ |
|-----------------------------|----------|---------|
| Rubber on concrete          | 1.0      | 0.8     |
| Steel on steel              | 0.74     | 0.57    |
| Aluminum on steel           | 0.61     | 0.47    |
| Glass on glass              | 0.94     | 0.4     |
| Copper on steel             | 0.53     | 0.36    |
| Wood on wood                | 0.25–0.5 | 0.2     |
| Waxed wood on wet snow      | 0.14     | 0.1     |
| Waxed wood on dry snow      | —        | 0.04    |
| Metal on metal (lubricated) | 0.15     | 0.06    |
| Teflon on Teflon            | 0.04     | 0.04    |
| Ice on ice                  | 0.1      | 0.03    |
| Synovial joints in humans   | 0.01     | 0.003   |

*Note:* All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

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## Friction in Newton's Laws Problems

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Friction is a force, so it simply is included in the  $\Sigma \vec{F}$  in Newton's Laws.

The rules of friction allow you to determine the direction and magnitude of the force of friction.

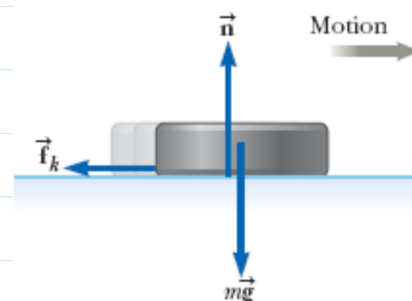
# The Sliding Hockey Puck

Saturday, 30 January, 2021 12:50

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

-  R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
-  J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY, 2014.
-  H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
-  H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A hockey puck on a frozen pond is given an initial speed of  $20 \text{ m/s}$ . If the puck always remains on the ice and slides  $115 \text{ m}$  before coming to rest, determine the coefficient of kinetic friction between the puck and ice.



## SOLUTION

Apply the particle under a net force model in the  $x$  direction to the puck:

$$(1) \sum F_x = -f_x = ma_x$$

Apply the particle in equilibrium model in the  $y$  direction to the puck:

$$(2) \sum F_y = n - mg = 0$$

Substitute  $n = mg$  from Equation (2) and  $f_k = \mu_k n$  into Equation (1):

$$-\mu_k n = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

Apply the particle under constant acceleration model to the puck,

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i), \text{ with } x_i = 0 \text{ and } v_f = 0:$$

$$0 = v_{xi}^2 + 2a_x x_f = v_{xi}^2 - 2\mu_k g x_f$$

Solve for the coefficient of kinetic friction:

$$\mu_k = \frac{v_{xi}^2}{2gx_f}$$

Substitute the numerical values:

$$\mu_k = \frac{20.0 \text{ m/s}^2}{2 \times 9.80 \text{ m/s}^2 \times 115 \text{ m}} = 0.177$$

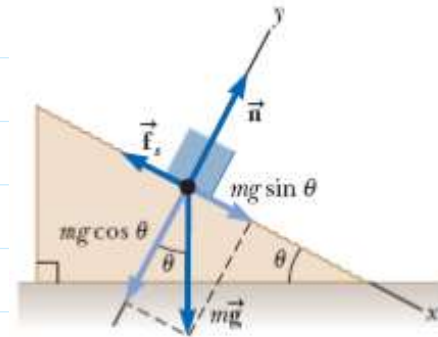
# Determination of $\mu_s$ and $\mu_k$

Saturday, 30 January, 2021 12:42

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

-  R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
-  J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY, 2014.
-  H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
-  H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Suppose a block is placed on a rough surface inclined relative to the horizontal as shown. The incline angle is increased until the block starts to move. Show that you can obtain  $\mu_s$  by measuring the critical angle  $\theta_c$  at which this slipping just occurs.



## SOLUTION

The diagram shows the forces on the block: the gravitational force  $m\vec{g}$ , the normal force  $\vec{n}$ , and the force of static friction  $\vec{f}_s$ . We choose  $x$  to be parallel to the plane and  $y$  perpendicular to it.

Apply the particle in equilibrium model to the block in both the  $x$  and  $y$  directions:

$$(1) \sum F_x = mg \sin \theta - f_s = 0$$

$$(2) \sum F_y = n - mg \cos \theta = 0$$

Substitute  $mg = n / \cos \theta$  from Equation (2) into Equation (1):

$$(3) f_s = mg \sin \theta = \left( \frac{n}{\cos \theta} \right) \sin \theta = n \tan \theta$$

When the incline angle is increased until the block is on the verge of slipping, the force of static friction has reached its maximum value  $\mu_s n$ . The angle  $\theta$  in this situation is the critical angle  $\theta_c$ . Make these substitutions in Equation (3):

$$\mu_s n = n \tan \theta_c$$

$$\mu_s = \tan \theta_c$$

Once the block starts to move at  $\theta \geq \theta_c$ , it accelerates down the incline and the force of friction is  $f_k = \mu_k n$ . If  $\theta$  is reduced to a value less than  $\theta_c$ , however, it may be possible to find an angle  $\theta'_c$  such that the block moves down the incline with constant speed as a particle in equilibrium again ( $a_x = 0$ ). In this case, use Equations (1) and (2) with  $f_s$  replaced by  $f_k$  to find  $\mu_k$ :  $\mu_k = \tan \theta'_c$ , where  $\theta'_c < \theta_c$ .

# F is greater than $f_s$

Saturday, 30 January, 2021 12:52

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

-  R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
-  J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY, 2014.
-  H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
-  H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A box weighing (100 N) is at rest on a horizontal floor. What is the acceleration of the box if a horizontal force of (40 N) is applied to it? ( $\mu_s = 0.34$  and  $\mu_k = 0.2$  )

The object will move only if:

$$F > f_{s,max}$$

$$f_{s,max} = \mu_s n = \mu_s mg = 0.34 \times 100 = 34 \text{ N}$$

$\therefore$  The object will move since:

$$F > f_{s,max}$$

$$\sum \vec{F}_x = m\vec{a} \Rightarrow F - f_k = ma \Rightarrow F - \mu_k n = ma \Rightarrow F - \mu_k mg = ma$$

$$\Rightarrow a = \frac{F - \mu_k mg}{m} = \frac{40 - 0.2 \times 100}{10} = 2 \text{ m/s}^2$$

## F is less than f<sub>s</sub>

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- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
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An object of mass (10 kg) is set on an inclined plane of angle ( $37^\circ$ ) with the horizontal. Find the force of friction between the surfaces (in units of N). ( $\mu_s = 0.9$  and  $\mu_k = 0.8$  )

The object is about to move when the angle of the inclined plane is:

$$\theta_c = \tan^{-1} \mu_s = \tan^{-1} 0.9 = 42^\circ$$

Since:  $\theta < \theta_c \Rightarrow$

The object will not move, and:

$$\sum \vec{F}_x = 0$$

$$\Rightarrow mg \sin 37 - f_s = 0$$

$$\Rightarrow f_s = mg \sin 37 = 60N$$

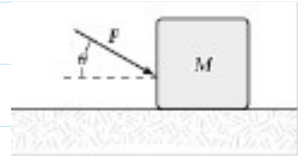
## F inclined below the horizontal

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Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

-  R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
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-  H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
-  H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A block is pushed across a horizontal surface by the force shown. If the coefficient of kinetic friction between the block and the surface is (0.3), ( $F = 20\text{ N}$ ), ( $\theta = 30^\circ$ ), and ( $M = 3\text{ kg}$ ). What is the magnitude of the acceleration of the block?



$$\sum \vec{F}_x = m\vec{a}$$

$$\Rightarrow F \cos 30 - f_k = ma$$

$$\Rightarrow F \cos 30 - \mu_k \hat{n} = ma$$

$$\Rightarrow F \cos 30 - \mu_k (Mg + F \sin 30) = ma$$

$$\Rightarrow 20 \cos 30 - 0.3 \times (3 \times 10 + 20 \sin 30) = 3 \times a \Rightarrow a = 1.8\text{ m/s}^2$$